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Center clusters and their percolation properties in lattice QCD

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Properties of local Polyakov loops are studied in finite temperature lattice QCD and SU(3) lattice gauge theory. We evaluate local Polyakov loops, identify the closest center element for each loop and investigate cluster properties of these center phases. For a suitable definition of the clusters we find that the deconfinement transition of pure SU(3) gauge theory may be characterized by percolation of the center clusters. For the case of full QCD cluster observables show a behavior which seems to be compatible with a smooth crossover type of transition.

The XXVIII International Symposium on Lattice Field Theory, Lattice2010
June 14-19, 2010
Villasimius, Italy

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1. Introduction

With the running and upcoming experiments at LHC, RHIC and GSI the deconfinement transition of QCD is currently a strong focus of research. We here report on our study of percolation aspects in the deconfinement transition of QCD and SU(3) gauge theory. Preliminary results were already presented in [1, 2]. Related studies for SU(2) gauge theory were reported in [3] – [6].

For analyzing the confinement-deconfinement transition the Polyakov loop may be used as an order parameter (for a recent alternative proposal see [7, 8]). We here distinguish between local Polyakov loops $L(\vec{x})$ located at a spatial point $\vec{x}$ and the spatially averaged loop $P = V^{-1}\sum_{\vec{x}} L(\vec{x})$, where V denotes the spatial volume. The local loop is given by the trace of the product of temporal gauge links $U_4(\vec{x},t)$ (N_t is the number of lattice points in time direction):

$$L(\vec{x}) \;=\; \mathrm{Tr} \prod_{t=0}^{N_t-1} U_4(\vec{x},t) \;, \tag{1.1}$$

i.e., the Polyakov loop is a gauge transporter that propagates a static quark at position $\vec{x}$ forward in time. The Polyakov loop is related to the free energy F_q of a single quark via $\langle L(\vec{x})\rangle = \langle P\rangle \propto \exp(-F_q/T)$, where T is the temperature. Below T_c the free energy is infinite, $\langle L(\vec{x})\rangle = \langle P\rangle = 0$, and the quarks are confined. Above T_c we have a finite free energy and thus $\langle L(\vec{x})\rangle = \langle P\rangle \neq 0$, signaling deconfinement. Thus the Polyakov loop acts as an order parameter for confinement.

The gauge group SU(3) has the three center elements $z = 1, e^{\pm i2\pi/3}$. A center transformation with a center element z transforms the temporal gauge links at a fixed time slice $t = t_0$: $U_4(\vec{x},t_0) \longrightarrow z\, U_4(\vec{x},t_0)$. While the measure and the gauge action are invariant under the center transformations, the Polyakov loop transforms non-trivially. A non-vanishing expectation value $\langle L(\vec{x})\rangle = \langle P\rangle \neq 0$ thus signals the spontaneous breaking of the center symmetry. This symmetry and its spontaneous breaking are at the core of the Svetitsky-Yaffe conjecture [9] which states that at T_c the system can be described by a 3D effective spin model with an action which is symmetric under the center group. The spin degrees of freedom are related to the local loops $L(\vec{x})$.

For such spin systems it is known that aligned spins form local clusters, and that at the critical temperature these clusters start to percolate. Various questions arise naturally:

- Can one identify such characteristic properties of spin systems directly in QCD?
- Are these properties important only at T_c, or in a finite interval of temperatures?
- What happens when one includes fermions which break the center symmetry explicitly?

2. Properties of local Polyakov loops

We study the distribution properties of the local Polyakov loop for quenched as well as dynamical SU(3) gauge configurations. In the quenched case we use the Lüscher-Weisz gauge action with lattice sizes from $20^3 \times 6$ to $40^3 \times 12$ and temperatures ranging from $T = 0.63\,T_c$ to $1.32\,T_c$ [1]. For the full theory the configurations have been produced by the Wuppertal-Budapest group, using a Symanzik improved gauge action and $2+1$ flavors of stout-link improved staggered quarks

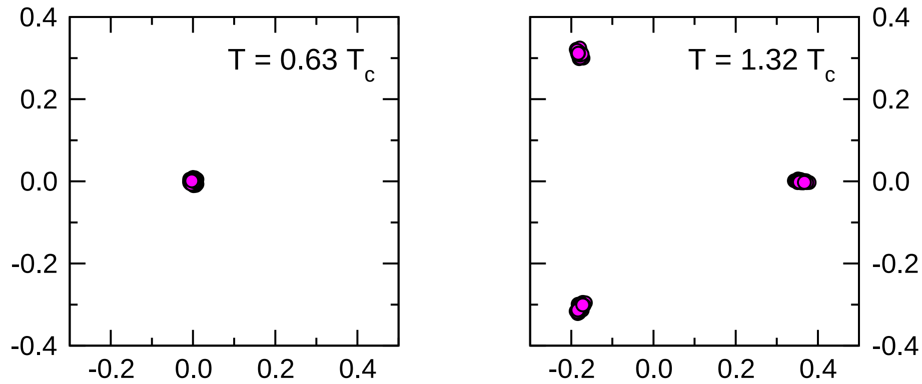

Figure 1: Scatter plot of the spatially averaged Polyakov loop P in the complex plane for pure gauge theory.

at physical quark masses [10]. The temperatures range from $T = 100$ MeV to 320 MeV with lattice sizes $18^3 \times 6, 24^3 \times 6, 36^3 \times 6, 24^3 \times 8$ and $32^3 \times 10$, such that finite volume as well as scaling studies can be performed.

Before we come to the analysis of the local loops $L(\vec{x})$ we briefly summarize the behavior of the spatially averaged loop P. This behavior is illustrated in Fig. 1, where we show scatter plots for the values of the Polyakov loop P in the complex plane. In the lhs. plot, which is for a temperature $T = 0.63\,T_c$, the values cluster near the origin, i.e., $\langle P \rangle = 0$. In the deconfined phase (rhs. plot, $T = 1.32\,T_c$) the values of P are non-vanishing. They scatter at angles 0 and $\pm 2\pi/3$, which reflects the underlying center symmetry that is broken spontaneously above T_c. In the infinite volume the system spontaneously selects one of the three center sectors and only the corresponding "island" in the complex plane is populated.

For the analysis of the distribution of the $L(\vec{x})$ we write the local loop as

$$L(\vec{x}) \;=\; \rho(\vec{x})\, e^{i\varphi(\vec{x})} \,. \tag{2.1}$$

We study the histograms $H[\rho(\vec{x})]$ of the distribution of the modulus $\rho(\vec{x})$, as well as the histograms $H[\varphi(\vec{x})]$ for the phase $\varphi(\vec{x})$. Analyzing the distribution of the modulus $\rho(\vec{x})$ we found that both, below and above T_c, for quenched as well as for full QCD the distribution is always the same and closely follows the distribution from Haar measure [1, 2]. From that finding we conclude that the relevant part of the information must be found in the phase $\varphi(\vec{x})$ of the local Polyakov loop.

The distribution of $H[\varphi(\vec{x})]$ is shown in Fig 2. In the quenched case we see that for temperatures below T_c we have peaks in the distribution of the phase at all three center values. These peaks are equally populated and average to zero ($1 + e^{i2\pi/3} + e^{-i2\pi/3} = 0$) when one calculates the spatially averaged Polyakov loop P. Above the phase transition the abundance of sites increases for one of the sectors in a process of spontaneous symmetry breaking. For full QCD the situation is similar. For very low temperatures we have peaks of almost equal height at all three center values. With increasing temperature the peak of the real sector starts to grow. This is because the fermion determinant acts like an external field that favors the real sector.

The question is now if the phase values at the spatial positions $\vec{x}$ are distributed independently of each other, or if they form spatial domains as is known from spin systems. To study this question we assign sector numbers $n(\vec{x})$ to the sites $\vec{x}$,

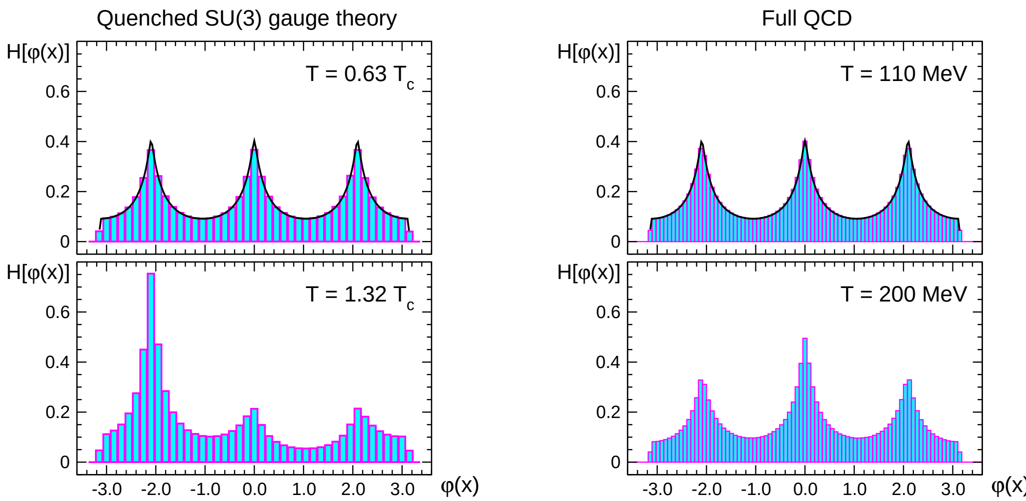

Figure 2: Histograms for the distribution of the local phase $\varphi(\vec{x})$. We compare quenched (lhs.) and full QCD (rhs.) at low and high T. The full curve is the Haar measure distribution $H(\varphi) = \int dU\, \delta(\varphi - \arg U)$.

$$n(\vec{x}) \;=\; \begin{cases} -1 & \text{for} \quad \varphi(\vec{x}) \in [-\pi+\delta\,,\, -\pi/3-\delta\,]\,, \\ \;\;\,0 & \text{for} \quad \varphi(\vec{x}) \in [-\pi/3+\delta\,, \pi/3-\delta\,]\,, \\ +1 & \text{for} \quad \varphi(\vec{x}) \in [\,\pi/3+\delta\,, \pi-\delta\,]\,. \end{cases} \tag{2.2}$$

If $\varphi(\vec{x})$ is in none of the three intervals no sector number is assigned to the site $\vec{x}$. δ is a free real and positive parameter which allows to cut lattice points $\vec{x}$ where the corresponding phase $\varphi(\vec{x})$ is near one of the local minima of the distributions in Fig. 2. The remaining lattice points $\vec{x}$ which survive the cut and are assigned a sector number $n(\vec{x})$ can now be organized in clusters. We put neighboring lattice sites $\vec{x},\vec{y}$ into the same cluster if $n(\vec{x}) = n(\vec{y})$.

As a first test we investigate the number of lattice points in the three possible center sectors as a function of temperature. In Fig. 3 we plot the occupation of the sectors for the quenched (lhs.) and dynamical data (rhs.) for $\delta = 0$, i.e., without any cut. We clearly see that in the quenched case the three sectors are equally populated below T_c. At the critical temperature one of the sectors increases its population while the other two sectors become depleted. For full QCD (rhs.) it is the real sector that increases its population above T_c due to the explicit symmetry breaking from the fermion determinant.

3. Cluster and percolation properties of center domains

We now study the dependence of the cluster size on the temperature and a possible percolation phenomenon of the clusters. For that we plot the number W of sites in the largest cluster ("weight of the cluster") normalized by the spatial volume V as a function of temperature (Fig. 4). For the quenched case (lhs. plot) the parameter δ was set for illustrative purposes such that 39% of the sites are cut. For full QCD a cut of 19% was used. The plots show that below T_c the clusters are finite with a fixed size, which leads to a volume dependence for the ratio W/V. Above T_c the largest cluster starts to percolate and fills a fixed fraction of the volume, as can be seen from the fact that now $\langle W/V \rangle$ is independent of V. For the quenched case the curves seem to approach a

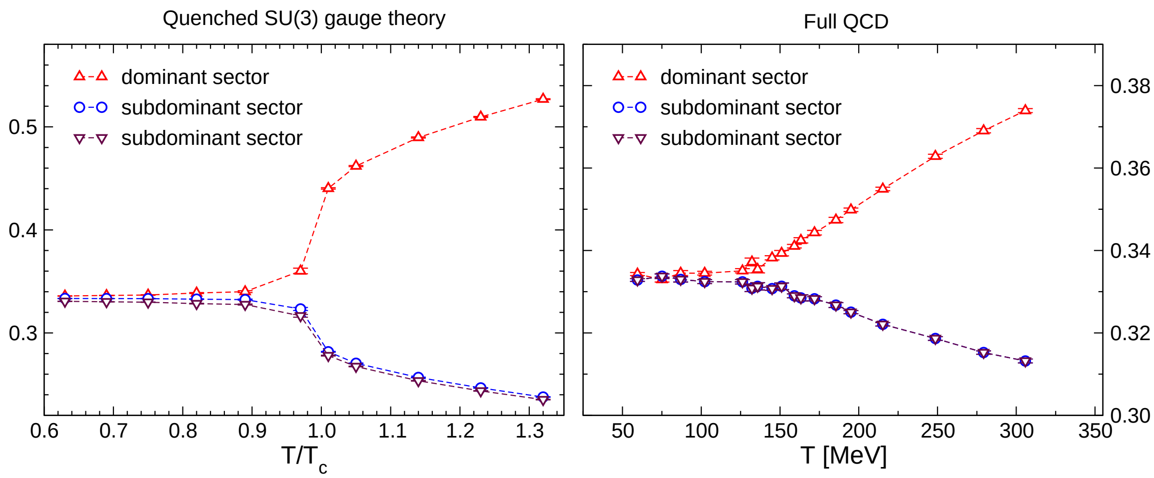

Figure 3: Abundance of lattice sites in the three center sectors as a function of temperature.

non-differentiable limit for $V \to \infty$, as expected for a first order transition. In the dynamical case a smooth behavior seems possible.

In Fig. 5 we plot the probability for finding a percolating cluster as a function of temperature. Again the quenched case (lhs.) seems to develop a step function, while for the dynamical percolation probability a smooth behavior seems feasible. For the latter case a more detailed finite volume analysis is necessary for a final conclusion on the behavior of the clusters near T_c.

4. Summary and outlook

In the project reported here we explore percolation aspects at the deconfinement transition of QCD and SU(3) lattice gauge theory. The starting point is an analysis of the phase of the local Polyakov loops which we find to cluster around the three center values. According to that phase we assign the lattice sites to center sectors and study the corresponding center clusters. For the quenched case we find a sharp onset of percolation of the center clusters at the deconfinement temperature T_c. For the case of full QCD the behavior seems to be more smooth and could be com-

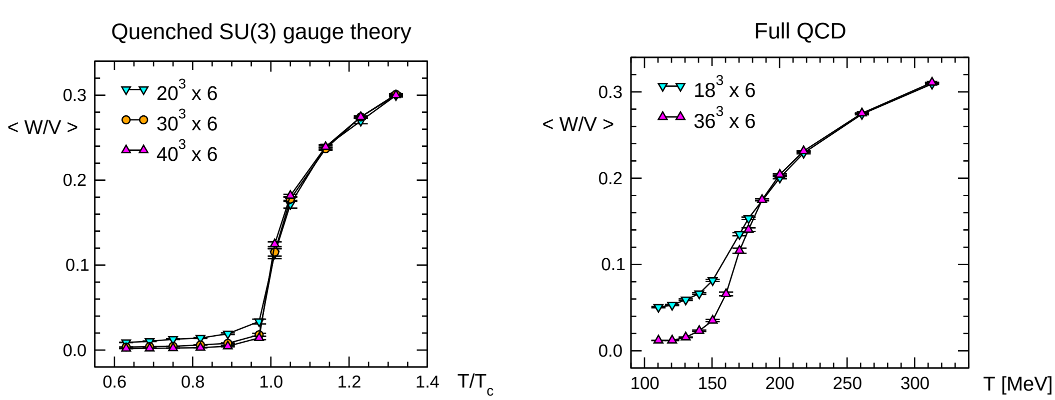

Figure 4: Weight W of the largest cluster normalized with the volume V as function of T.

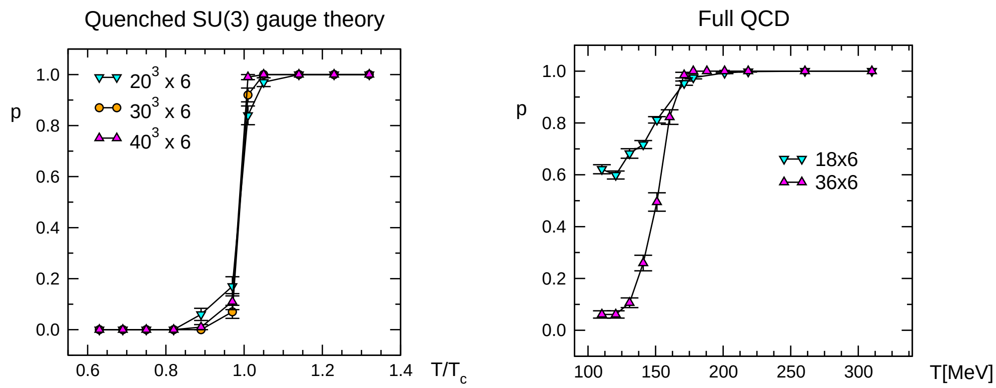

Figure 5: Percolation probability p as a function of T.

patible with the crossover type of transition expected for full QCD. Additional finite size studies will, however, be necessary to clearly establish that behavior.

An important question is whether the clusters and their percolation properties can be given a precise meaning in the continuum limit. We have begun to study this question by comparing results on lattices with different lattice constant a. The cluster definition, i.e., the parameter δ, is set such, that below T_c the clusters have a fixed diameter in physical units. This is repeated for several values of a and the flow of the parameters is monitored. First results indicate that indeed a continuum limit of the clusters and the percolation picture seems possible [11].

Acknowledgments: We thank Christian Lang, Axel Maas and Alexander Schmidt for interesting discussions. This work was partly supported by the DFG SFB TR 55 and the FWF DK 1203.

References

[1] Gattringer C., 2010, *Phys.Lett.B* **690** 179.

[2] Borsanyi S., Danzer J., Fodor Z., Gattringer C., Schmidt A., 2010, arXiv:1007.5403

[3] Fortunato S., and Satz H., 2000, *Phys. Lett. B* **475** 311; *Nucl. Phys. A* **681** 466.

[4] Fortunato S. et al., 2001, *Phys. Lett. B* **502** 321.

[5] Fortunato S., 2003, *J. Phys. A* **36** 4269.

[6] Schmidt A., 2010, Master Thesis, Karl-Franzens Universitẗ, Graz, Austria, `http://physik.uni-graz.at/itp/files/schmidt/masterthesis.pdf`

[7] Gattringer C., 2006, *Phys. Rev. Lett.* **97** 032003; Bilgici E. et al, 2010, *Few Body Syst.* **47** 125; 2008, *Phys. Rev. D* **77** 094007.

[8] F. Synatschke, A. Wipf and C. Wozar, 2007, Phys. Rev. D **75** 114003; F. Synatschke, A. Wipf and K. Langfeld, 2008 Phys. Rev. D **77** 114018.

[9] Yaffe L.G., Svetitsky B., 1982, *Phys. Rev. D* **26**, 963; *Nucl. Phys. B* **210** 423.

[10] Aoki Y., et al., 2006, *JHEP* 0601:089; *Phys. Lett. B* **643** 46; Aoki Y., et al., 2009, *JHEP* 0906:088; Aoki Y., et al., 2006, *Nature* 443:675-678.

[11] Work in preparation.